Experimental manifestations of duality in KN interactions and implications for the bootstrap



1974 J. Phys. A: Math. Nucl. Gen. 7 1135

(http://iopscience.iop.org/0301-0015/7/10/005)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 10, 1974. Printed in Great Britain. 

# Experimental manifestations of duality in $\bar{K}N$ interactions and implications for the bootstrap

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Received 7 March 1974

**Abstract.** We suggest that various channel cross sections for $K^-p$ interactions strongly exhibit their different duality properties in the resonance region as well as at high energies. In particular, it is argued that non-diffractive processes with no $(u, t)$ dual amplitude are resonance dominated in the real part as well as the imaginary part. Thus, from a bootstrap point of view, the $(u, t)$ amplitude should be considered the driving force behind the $(s, t)$ and $(s, u)$ amplitudes and reciprocally.

$\bar{K}N$ interactions provide a very fruitful ground for testing duality predictions for the simple reason that their SU(3) properties forbid certain dual amplitudes from contributing to certain channels. This is very neatly summarized by the duality diagrams of figure 1 (Harari 1969, Rosner 1969), in which $(s, t)$, $(u, t)$ and $(s, u)$ dual amplitudes are described by diagrams 1($a$), 1($b$) and 1($c$) respectively.

Assuming that these amplitudes represent some first approximation, to be modified only by absorptive corrections (which in some cases, such as elastic scattering, may be very large) we find that for the well measured processes of table 1 only the diagrams indicated can contribute. (We assume that the lines are conventional quarks and that

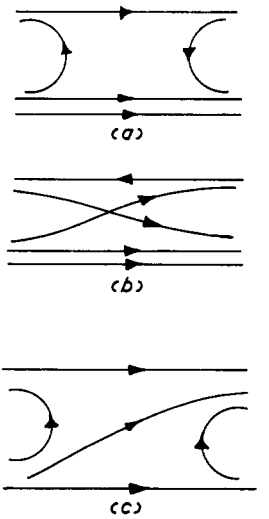


**Figure 1.** SU(3) duality diagrams.

**Table 1.** Allowed diagrams for various reactions.

| Process | Allowed diagrams of figure 1 |
|---|---|
| $K^-p \to K^-p$ | (*a*) |
| $K^-p \to \bar{K}^0n$ | (*a*) |
| $K^-p \to \pi^0\Lambda$ | (*b*), (*c*) |
| $K^-p \to \pi^-\Sigma^+$ | (*b*), (*c*) |
| $K^-p \to \pi^+\Sigma^-$ | (*c*) |
| $K^+p \to K^+p$ | (*b*) |

no strange quarks are present in the pion.) This situation is quite unlike $\pi N \to \pi N$ where all three diagrams contribute to all charge states. Thus it seems reasonable that the difference between these amplitudes will manifest themselves in $\bar{K}N$ data, but not in $\pi N$ data where all channels will have a similar structure.

Some evidence of this is already well known.

(i) At high energies, amplitudes with more quarks exchanged fall off quicker with energy. An interesting summary of this has been given by Lo (1972).

(ii) Resonances in 1(*a*) add up at fixed $t$ and cancel at fixed $u$. The opposite occurs in 1(*c*). This has been well summarized for $\bar{K}N$ interactions by Schmid and Storrow (1971) (see also Bricman *et al* 1971).

The purpose of this paper is to show how the properties of 1(*b*) and further properties of 1(*a*) and 1(*c*) also show up in the data.

It is well known that $K^+p$ interactions (pure 1(*b*)) are empirically structureless except for threshold effects. Thus it is usually postulated that 1(*b*) has no direct channel resonances.

Exchange degeneracy in the $t$ channel suggests that it should be purely real (apart from absorptive corrections).

We shall argue here that this real background in 1(*b*) is much larger than any real background in 1(*a*) and 1(*c*). Equivalently, in 1(*a*) and 1(*c*) the real part is resonance dominated as well as the imaginary part.

This idea is certainly suggested by the data. In figure 2 we have sketched the structure of the $K^-p$ channel cross sections from the CERN/HERA compilation (Bracci *et al* 1972). It is quite clear that in $K^-p \to \bar{K}^0n$ (pure 1(*a*)) and $K^-p \to \pi^+\Sigma^-$ (pure 1(*c*)) the resonances stand well clear of any background. In $K^-p \to K^-p$ there is clearly a large background (presumably pomeron in origin) and in $K^-p \to \pi^0\Lambda$, $\pi^-\Sigma^+$ there is also a large background which tends to swamp the resonance structure. It is therefore quite natural to identify this background almost completely with diagram 1(*b*). (The reader should note that although individual resonances should couple equally to $\pi^-\Sigma^+$ and $\pi^+\Sigma^-$ the absence of peaks in $K^-p \to \pi^-\Sigma^+$ compared to $\pi^+\Sigma^-$ can be explained by destructive interference. However, even without resonance peaks, the $\pi^-\Sigma^+$ cross section is still as high as the top of the $\pi^+\Sigma^-$ peak at the same energy indicating a background term as strong as the resonances.)

Sceptics will suggest, of course, that this 'background' is in fact due to hidden resonances which would only show up in phase shift analysis. Thus we have taken the numerical predictions of Lea *et al* (1973)† from their multi-channel $\bar{K}N$ analysis and

† We have used their corrected numerical values which differ slightly from those published.

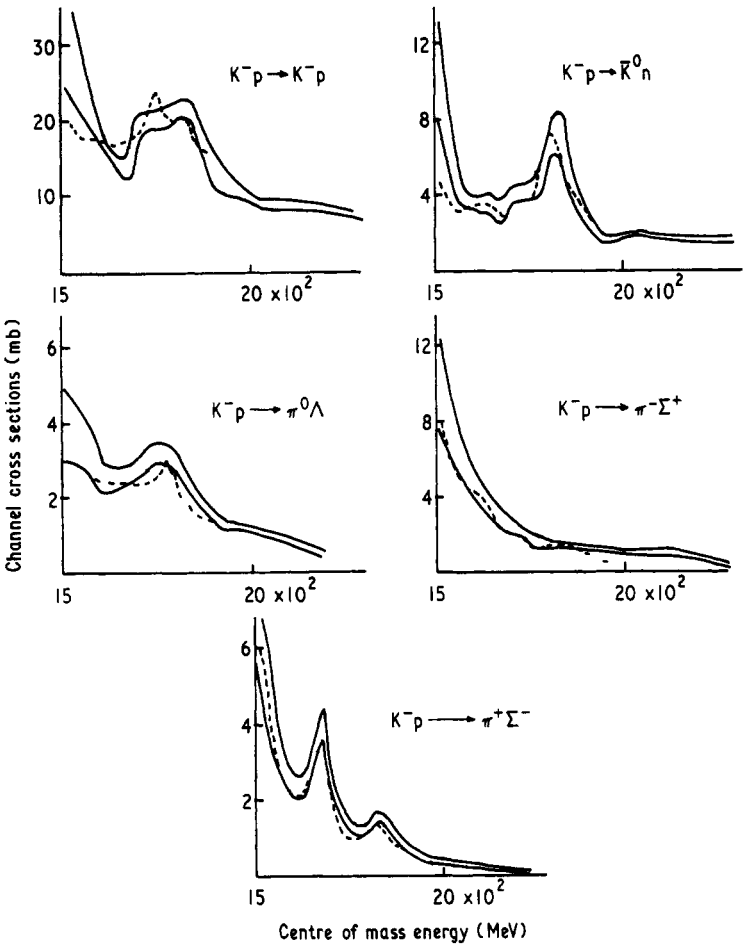


**Figure 2.** Channel cross sections for various $K^-p$ reactions. The full curves represent an eyeball limitation of the data points and their errors (from Bracci *et al* 1972). The broken curve is the prediction from the fit of Lea *et al* (1973).

constructed the s wave $\bar{K}N \to \pi\Sigma$ Argand plots for the two charge states. (Only their s wave plots have a large background as would be expected from something with a crossed-channel origin at low energies.)

These plots are shown in figure 3. It is quite clear that a large real background has shifted the $\pi^-\Sigma^+$ circles to the right, whereas the $\pi^+\Sigma^-$ plots are centrally situated about the imaginary axis.

Having verified for this case that 1(*a*) and 1(*c*) are resonance dominated in the real part as well as the imaginary part, whereas 1(*b*) has a large real background, we must now explain this fact. The naive duality argument is simple. In 1(*b*) the real background comes purely from crossed-channel contributions whereas in 1(*a*) and 1(*c*) the crossed-channel contributions are already included in the direct-channel resonances. However, duality statements such as these are usually applied only to the imaginary part in some simple form such as an FESR. For instance, 1(*c*) is expected to obey the FESR (assuming

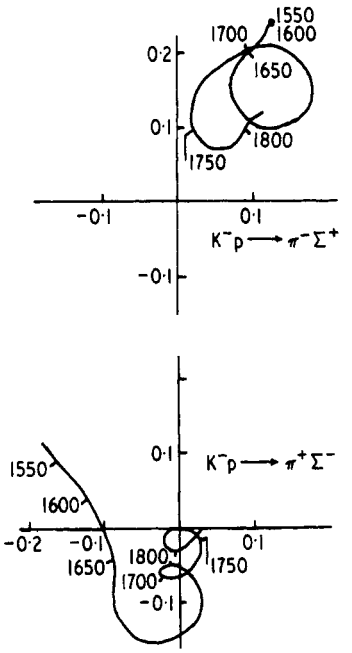


**Figure 3.** $S_{1/2}$ Argand plots from the predictions of Lea *et al* (1973) in the range 0·4 → 1·2 GeV/*c*. Centre of mass energies are shown in MeV.

no fixed pole contributions):

$$\int_{s_0}^{s} \text{Im } A(s', t)\, ds' \simeq 0 \tag{1}$$

$$\int_{u_0}^{u} \text{Im } A(u', t)\, du' \simeq 0 \tag{2}$$

$$\int_{s_0}^{s} [\text{Im } A(s', u) - \text{Im } R(s', u)]\, ds' \simeq 0 \tag{3}$$

$$\int_{u_0}^{u} [\text{Im } A(u', s) - \text{Im } R(u', s)]\, du' \simeq 0. \tag{4}$$

Equation (1) contains the property (ii) above. The cross-channel terms enter into equations (2), (3) and (4). In the forward half of the scattering plane we can evaluate the amplitude by an unsubtracted fixed $t$ dispersion relation. Equation (2) indicates that the left-hand cut is self-cancelling. Thus in the $s$ channel physical region, the various contributions to the real part from the left-hand cut integral will also tend to cancel. Thus the only strong contribution will come from the direct-channel resonances. (To get an idea of how far we need to go into the $s$ channel physical region for these cancellations to occur, note that the resonances which cancel each other in equation (1) and (2) are typically separated by about 1·0 $(\text{GeV})^2$ and that this is of the same order as the separation of left- and right-hand thresholds.)

In the backward direction, we use an unsubtracted fixed $u$ dispersion relation. Then there is no left-hand cut contribution at all, since the $t$ channel is non-resonant.

Thus it would seem plausible that the $(s, u)$ dual amplitude should be resonance dominated in both real and imaginary parts in the $s$ and $u$ channel physical regions. But what of the $t$ channel physical region? Whether we use a fixed $u$ or fixed $s$ dispersion relation we will get a definite non-zero real part contributing at $t > t_0$ because equations (3) and (4) show that the imaginary part does not average around zero in these left-hand cuts but gives a definite Regge-like structure.

Thus we can summarize the major qualitative features of the $(s, u)$ dual amplitude (figure 1(*c*)) as in figure 4, and similarly for the $(s, t)$ and $(u, t)$ amplitudes. These features are contained in the Veneziano model (Veneziano 1968) and have been shown consistent with data in $\pi$N interactions (Coulter *et al* 1969).

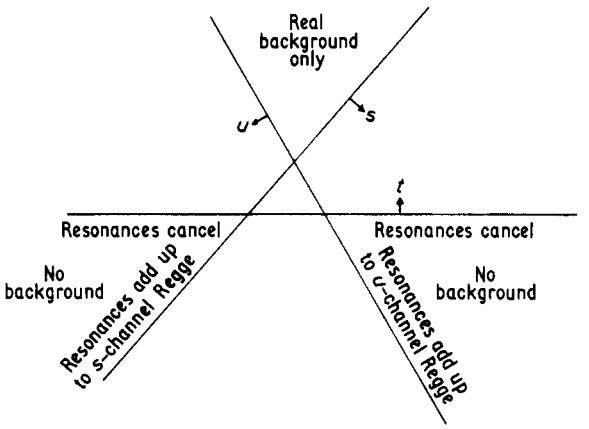


**Figure 4.** Qualitative features of $(s, u)$ dual amplitude in Mandelstam plane.

So far, we have not discussed in detail how to separate, theoretically, the resonances from background in the real part. Figure 3 indicates that it is certainly reasonable to do so. However, to perform this theoretical separation needs a specific model and models for resonances are infamous for their variety of forms for the real part. We contend that as long as the real part is evaluated from the imaginary part via a dispersion relation this ambiguity will be small, and that the reason for the ambiguities in real parts is simply because modellers have *not* distinguished between direct-channel resonances and the background coming from crossed-channel singularities, which 'drives' the resonances. In our approach these 'force' terms are contained in the $(u, t)$ amplitude only. This suggests a new approach to the bootstrap. Starting with one dual amplitude (say $(u, t)$) we could use this to generate the $(s, t)$ and $(s, u)$ amplitudes, from analyticity and unitarity. Relations between resonance couplings would follow from analyticity and the sum rules (or the unitarity limit on the real part (Lyth and York 1972) of partial-wave amplitudes) and masses would be determined from multi-channel unitarity. Clearly, from the cyclic symmetry, it does not matter which amplitude we start with, thus reciprocity is assured.

## Acknowledgments

I would like to thank G H Burkhardt and G P MacCauley for many conversations, C Michael for useful criticism and B R Martin for communicating the results of Lea *et al* (1973).

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